October 29, 2014

ECOS-EURISOL Joint Town Meeting - Orsay

New ideas and developments on EDF theory

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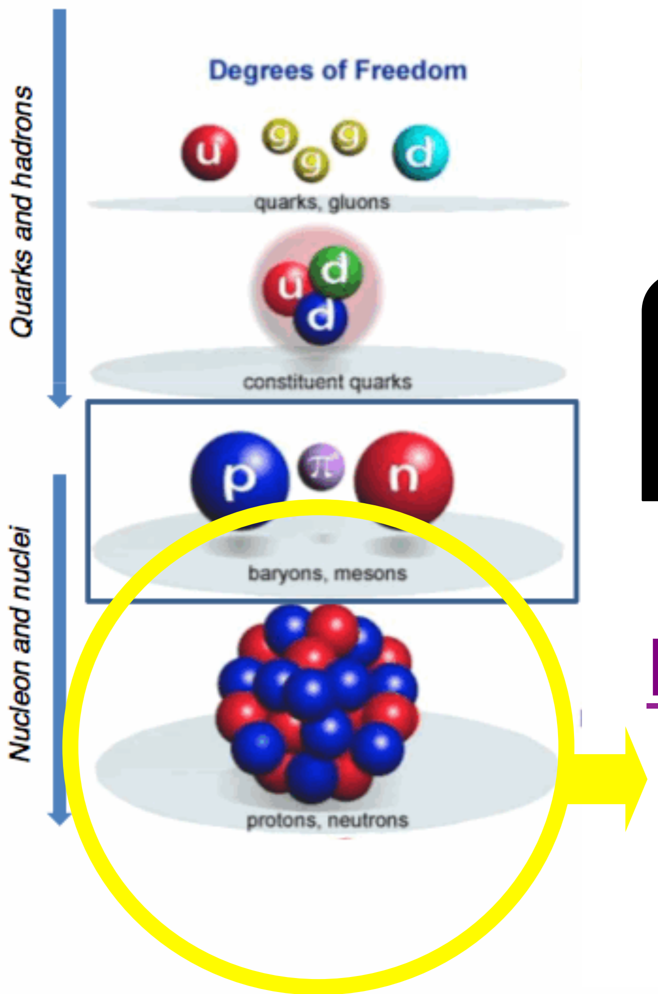
STRONGLY INTERACTING SYSTEMS



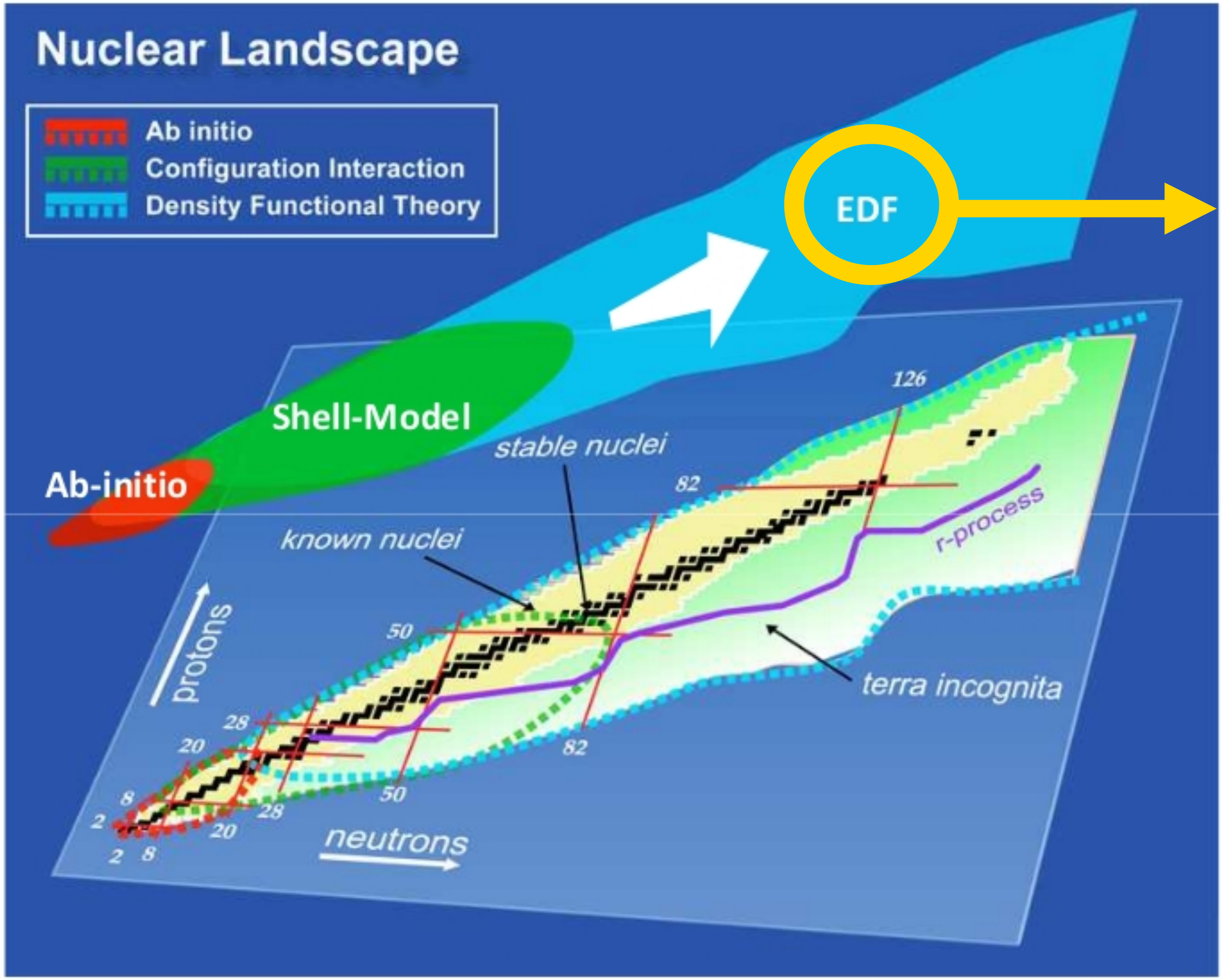
Quark-gluon dynamics

By neglecting the internal degrees of freedom of nucleons

Low-energy nuclear physics. Nucleus treated as a many-body system composed by nucleons

A unified theory for nuclear structure, reactions and stars

The Energy Density Functional (EDF) concept



http://unedf.org/

Mean field

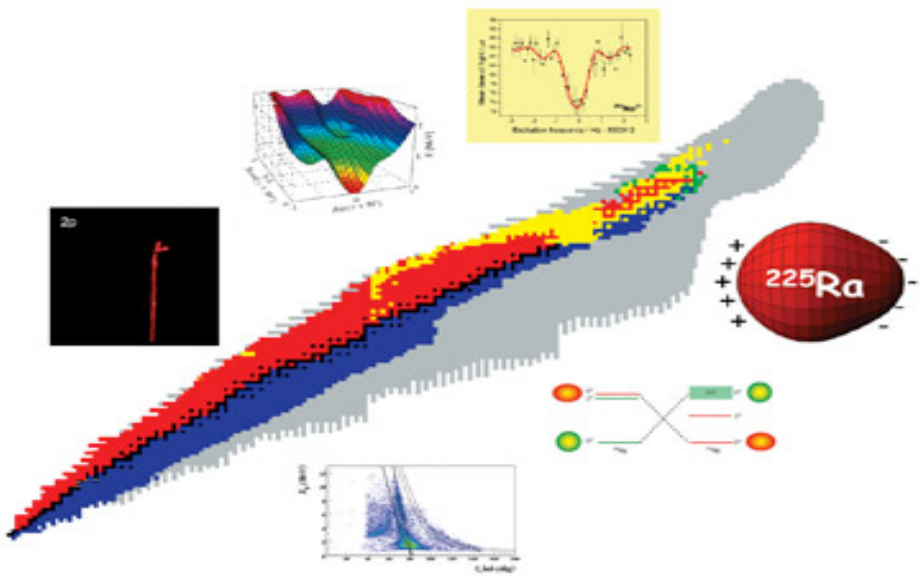
- Ground-state nuclear structure (radii, masses, deformations.)

- Low- and high-lying excitations (small-amplitude oscillations)

- Beyond small-amplitude oscillations: time-dependent mean field for dynamics

Typically: phenomenological interactions adjusted with mean-field calculations

Mean-field models are not always accurate enough / do not contain enough correlations …



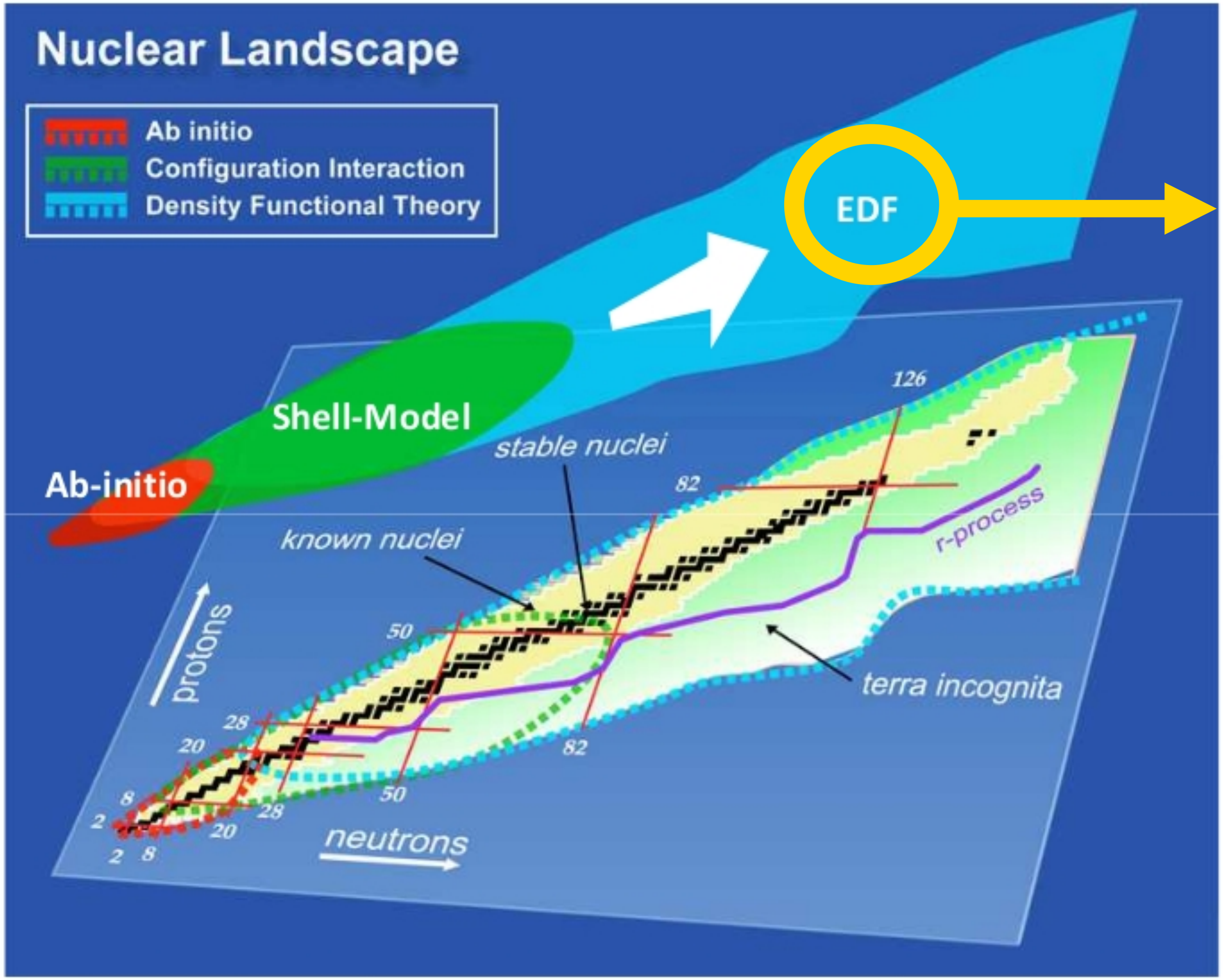
Necessity of implementing the theoretical framework to explicitly include more correlations:

configuration mixing, coupling between different degrees of freedom, ….

For example to analyze spectroscopic properties (and their evolution far from stability)

A unified theory for nuclear structure, reactions and stars

The Energy Density Functional (EDF) concept



http://unedf.org/

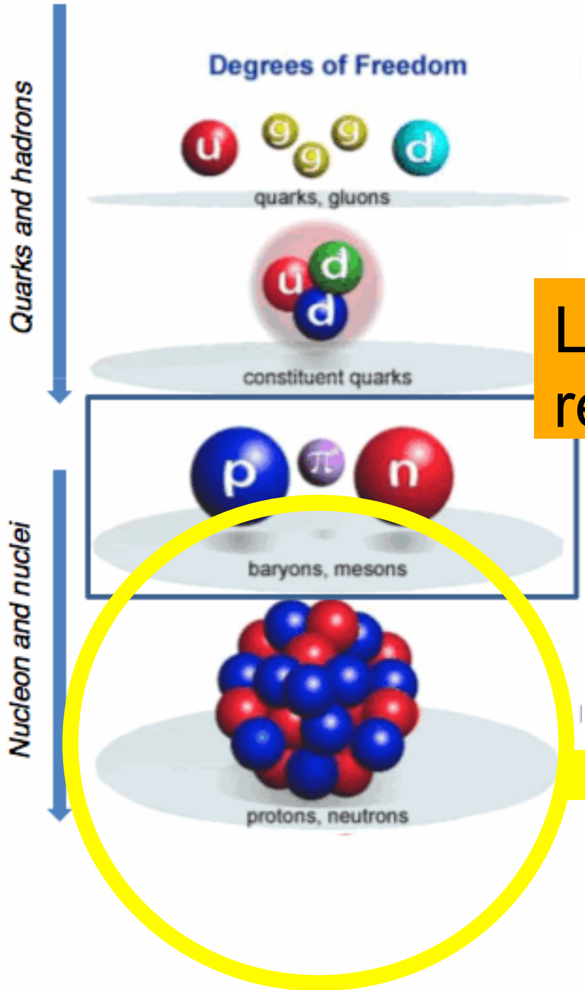
Beyond-mean -field models (correlations).

- Describing complex phenomena

- Improving predictive power of models

- ***NUMERICAL COMPLEXITY***
- ***DIVERGENCES***
- ***INTERACTION ?***

STRONGLY INTERACTING SYSTEMS



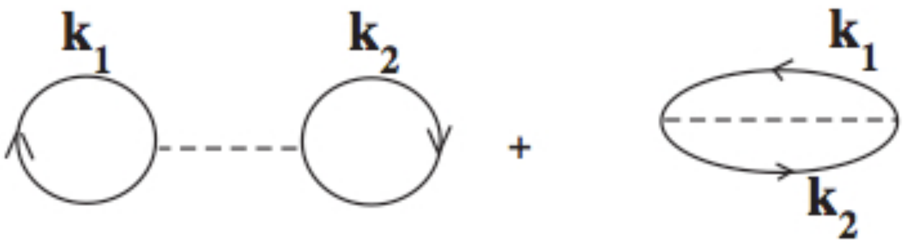
Quark-gluon dynamics

Links through EFT (nuclear interaction, regularization techniques, …)

Beyond mean-field methods

The mean-field approximation represents the leading order of the perturbative many-body problem.

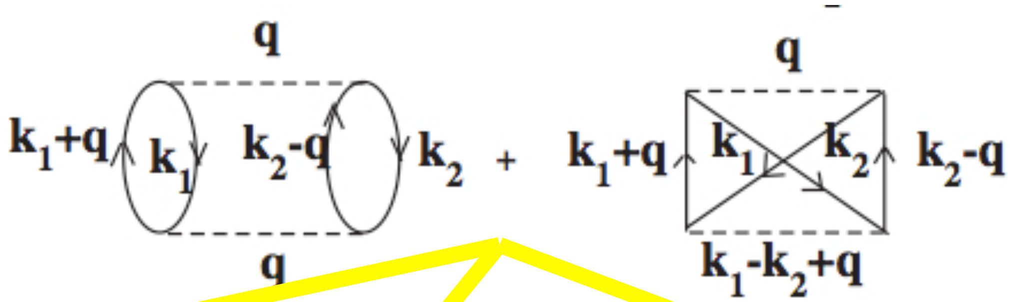
Total energy at the first order



For example: to calculate the 1st order equation of state of matter

What happens if one goes beyond the mean-field level within the EDF framework?

For example, the 2nd order for the equation of state of nuclear matter:



1

Commonly used interactions (Skyrme) are adjusted at the mean-field level. Double counting problems when one uses them beyond

2

Zero-range terms in conventional forces -> ultraviolet divergences when one goes beyond the mean field

3

Is our problem really perturbative when conventional forces are used?

Regularization techniques

Power counting analysis

Effective field theories (chiral interactions)

Last years, in Orsay…

1

Commonly used interactions (Skyrme) are adjusted at the mean-field level. **Double counting** ***problems when one uses them beyond***

2

Zero-range terms in conventional forces -> **ultraviolet divergences** ***when one goes beyond the mean field***

… these two items have been addressed:

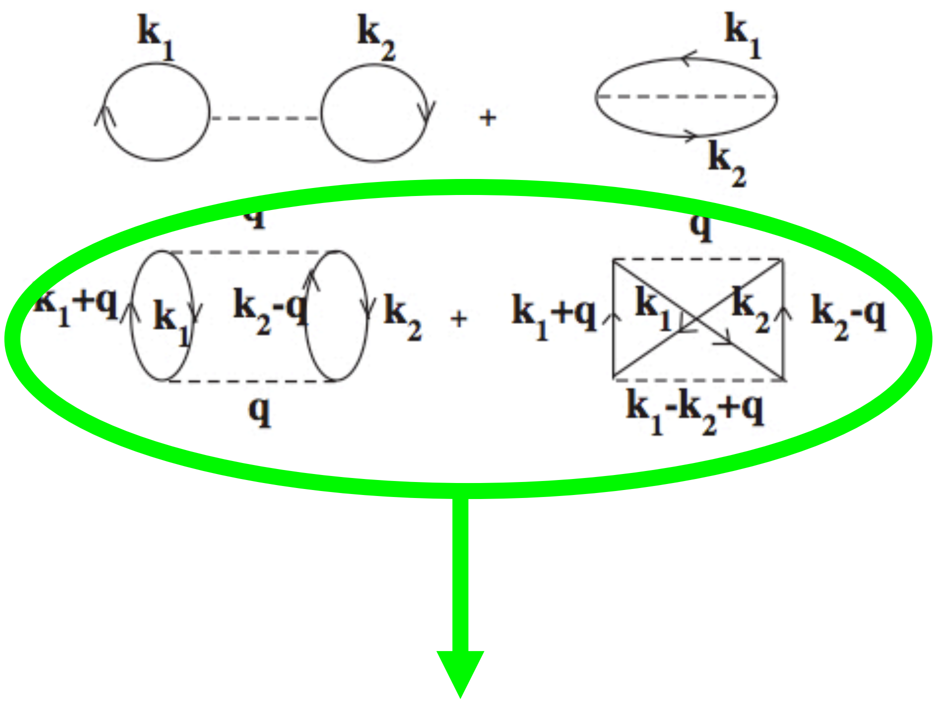
- **PhD thesis Kassem Moghrabi, Orsay**
- **Moghrabi, Grasso, Colò, Van Giai, PRL 105, 262501 (2010)**
- **Moghrabi, Grasso, Roca-Maza, Colò, PRC 85, 044323 (2012)**
- **Moghrabi and Grasso, PRC 86, 044319 (2012)**
- **Moghrabi, Grasso, van Kolck, arXiv:1312.5949**

PRL 105, 262501 (2010) PHYSICAL REVIEW LETTERS week ending 31 DECEMBER 2010

Beyond Mean-Field Theories with Zero-Range Effective Interactions: A Way to Handle the Ultraviolet Divergence

K. Moghrabi,[1,2] M. Grasso,[1] G. Colò,[3] and N. Van Giai[1]

Equation of state of nuclear matter with a Skyrme-type interaction



This second-order contribution diverges with a Skyrme-type interaction

Asymptotic behavior: linear divergence (with respect to the cutoff). The second-order correction is proportional to:

$$\frac{-11+2\ln 2}{105}+\frac{\Lambda}{9k_F}-\frac{2k_F}{45\Lambda}+O\left(\frac{k_F^2}{\Lambda^2}\right)$$

How the equation of state looks like:

Two problems: divergence and double counting!!!

First strategy: cutoff regularization

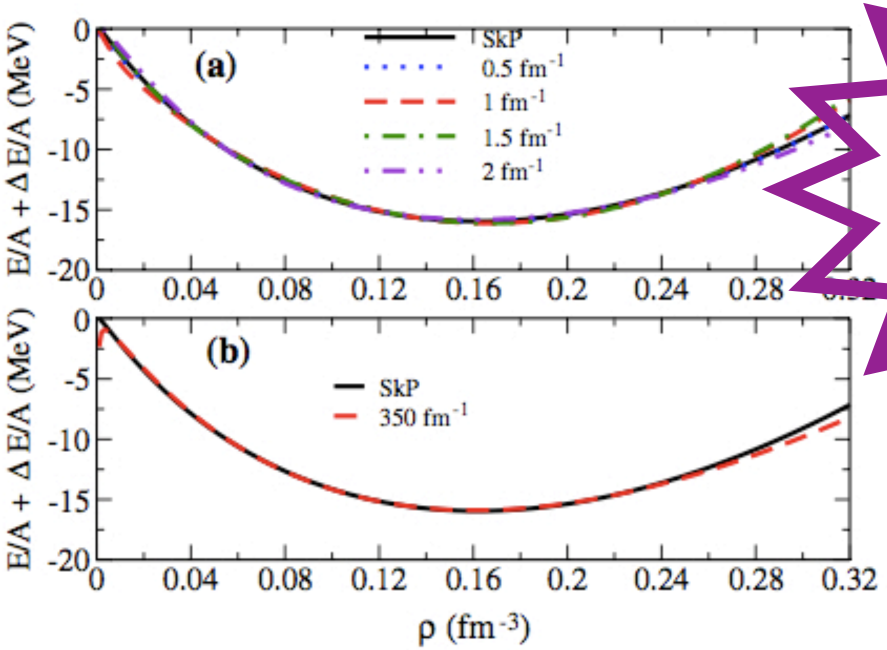
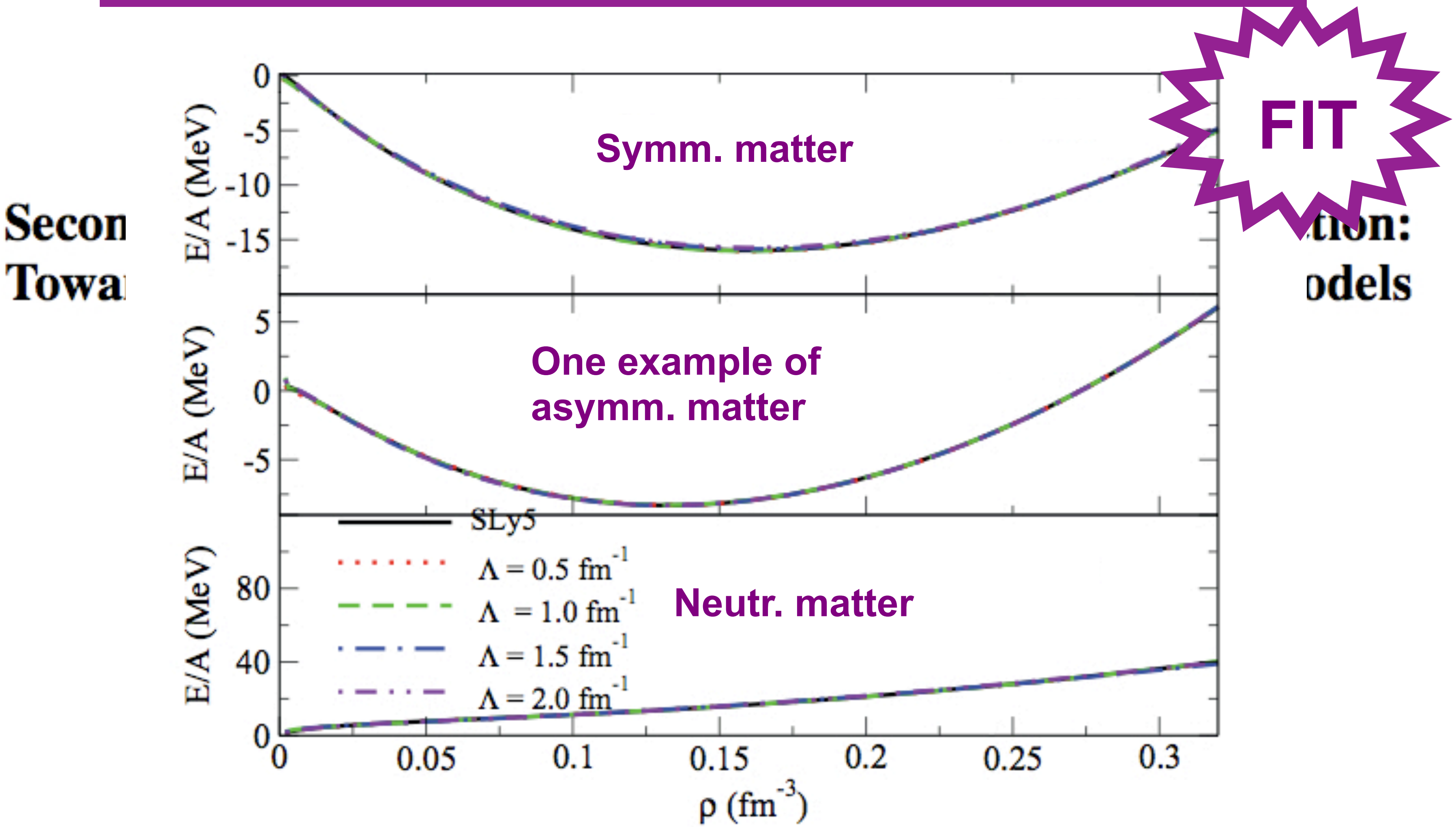


FIG. 4 (color online). (a) Second-order-corrected equations of state compared with the reference equation of state (SkP at mean-field level). (b) Extreme case of $\Lambda = 350$ fm^{-1}.

FIT: for each cutoff value

Adding the velocity-dependent part of the Skyrme interaction and treating also asymmetric nuclear matter

Again a cutoff regularization strategy

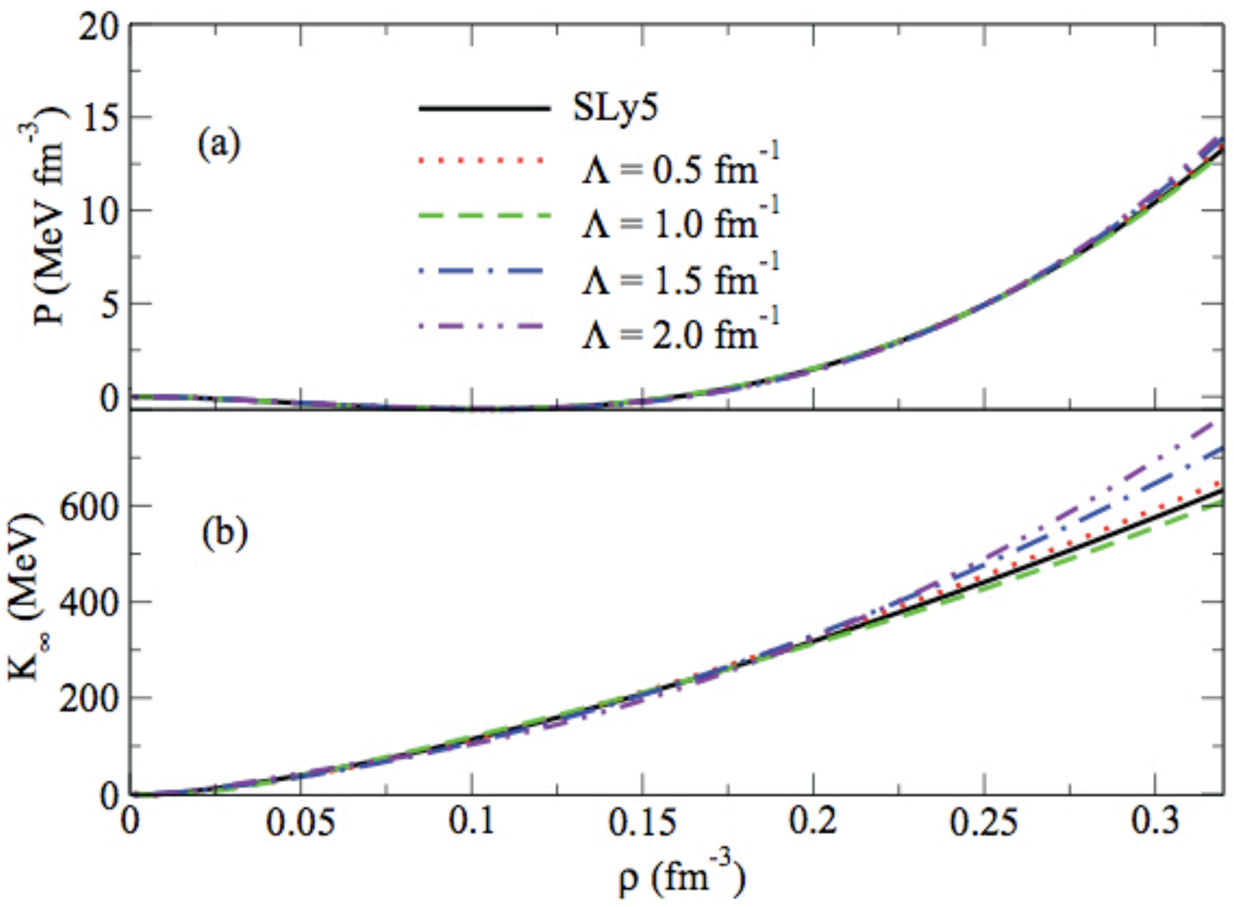


Stronger divergence

$$\frac{\Delta E^{(2)}}{A}(\delta, \rho, \Lambda \to \infty) = a^1_{\delta,\rho}\Lambda^5 + a^2_{\delta,\rho}\Lambda^3 + a^3_{\delta,\rho}\Lambda + a^4_{\delta,\rho} + O\left(\frac{k_F}{\Lambda}\right)$$

$$\delta = \frac{\rho_n - \rho_p}{\rho_n + \rho_p}$$

Pressure and Incompressibility do not enter in the fit. Check:



Pressure

Incompressibility

Pressure (a) and incompressibility (b) evaluated with the parameters obtained with the global fit.

… towards an interface with EFT

1

Commonly used interactions (Skyrme) are adjusted at the mean-field level. Double counting *problems when one uses them beyond*

2

Zero-range terms in conventional forces -> ultraviolet divergences *when one goes beyond the mean field*

Regularization techniques. Dimensional regularization

Effective field theories (chiral interactions)

PHYSICAL REVIEW C **86**, 044319 (2012)

Dimensional regularization applied to nuclear matter with a zero-range interaction

Kassem Moghrabi and Marcella Grasso

This procedure has been introduced in the framework of electroweak theories and consists of replacing the dimension of the divergent integrals with a continuous variable d.

…

One then performs a kind of analytic continuation in the dimension to return to the initial integer values.

…

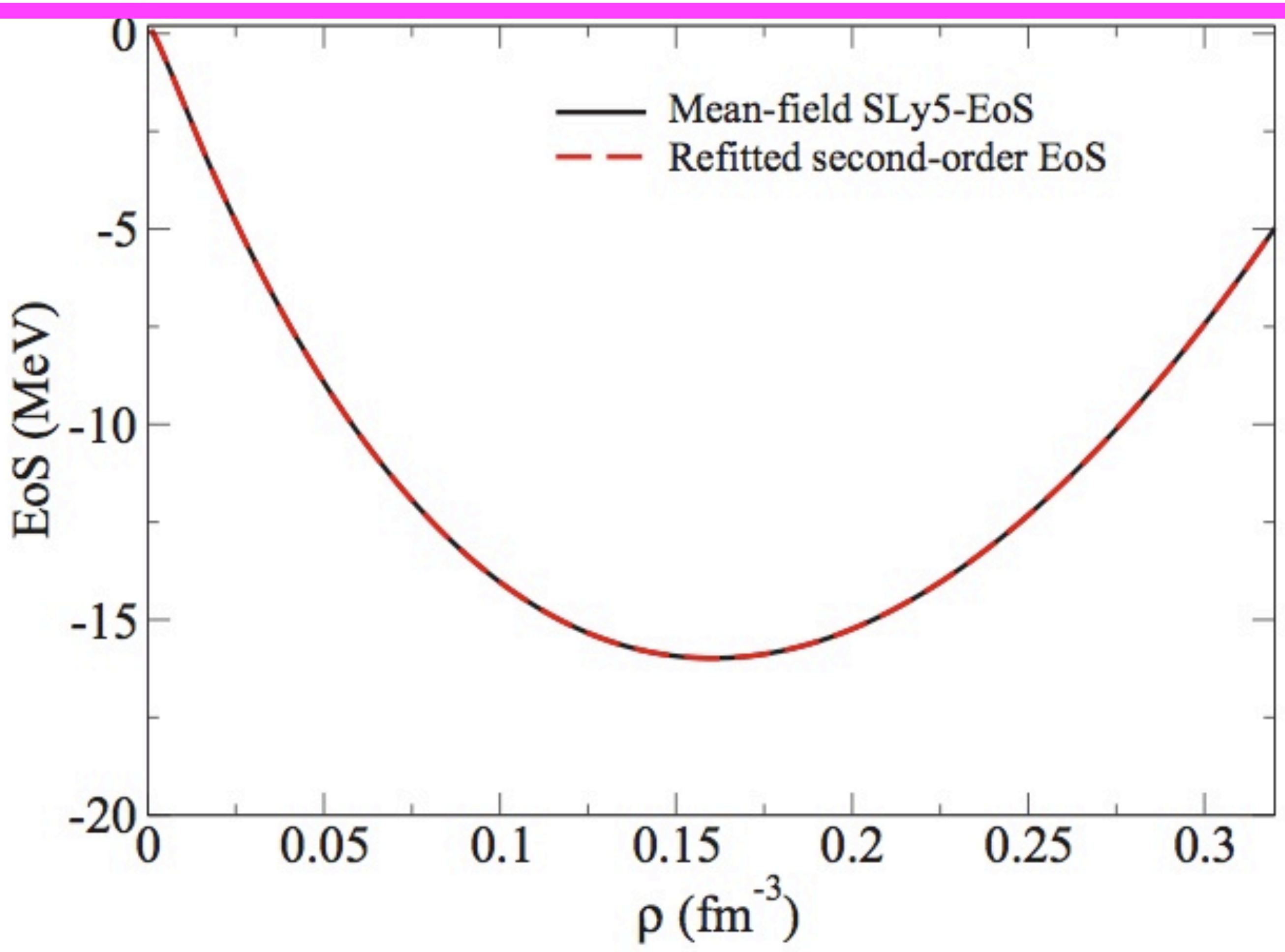
The dimensional regularization eliminates power-law divergences

…

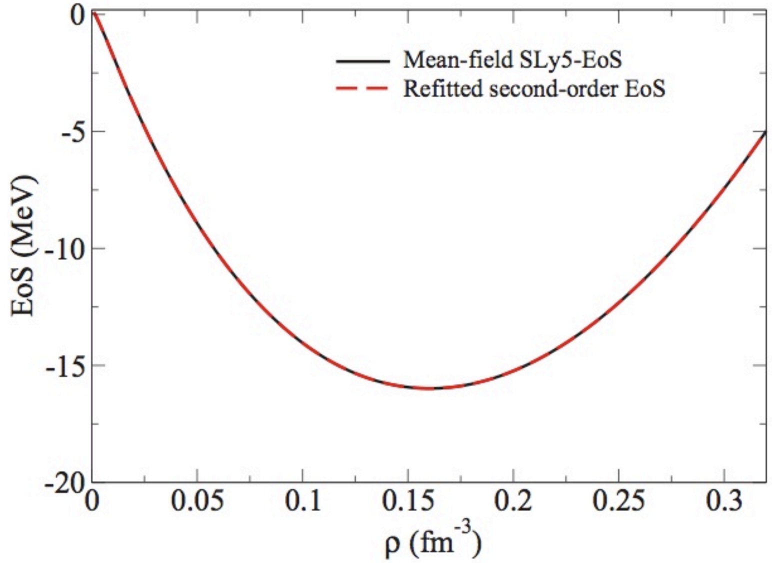
A regulator ε is introduced (when ε->0 the dimension of the integral goes back to the integer value). An auxiliary scale μ is introduced to maintain the correct dimensions of the physical quantities.

The fit is performed now to handle the problems of double counting. A unique set (no cutoff dependence) ->

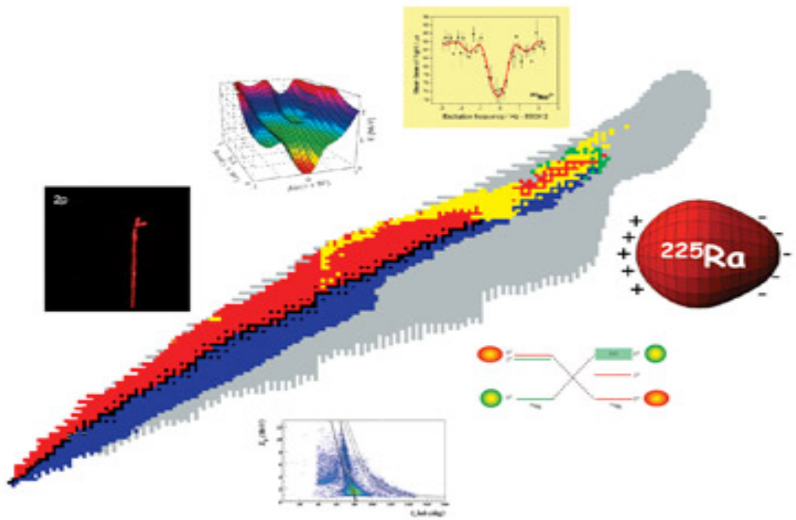
a unique beyond-mean-field interaction



Open problem … to be addressed in future



Going from matter …



… to finite nuclei with beyond-mean-field models. First attempt:

Brenna, Colo, Roca-Maza, arXiv: 1410.1302

… by keeping an EFT way of analyzing the problem…

Is our problem renormalizable with the Skyrme force?

arXiv:1312.5949

Renormalizability of the Nuclear Many-Body Problem with the Skyrme Interaction Beyond Mean Field

K. Moghrabi,[1] M. Grasso,[1] and U. van Kolck[1]

Renormalizability means that the theory is independent of the regularization (observables are independent of the cutoff)

The objective is ro reveal the implications of demanding renormalizability through a redefinition of the existing Skyrme parameters at each order

$$\frac{E}{A}(k_F,\Lambda) = \frac{3\hbar^2}{10m}k_F^2 + \frac{t_0}{4\pi^2}k_F^3 + \frac{T_3}{24\pi^2}k_F^{3+3\alpha} + \frac{\Theta_s}{40\pi^2}k_F^5 + \frac{\Delta E^{(2)}}{A}(k_F,\Lambda).$$

Finite

Absorbed

$$\frac{\Delta E^{(2)}(k_F,\Lambda)}{A} = \frac{\Delta E_f^{(2)}(k_F)}{A} + \frac{\Delta E_a^{(2)}(k_F,\Lambda)}{A} + \frac{\Delta E_d^{(2)}(k_F,\Lambda)}{A},$$

Divergent

An EFT way of analyzing the problem…

Is our problem renormalizable?

$$\frac{E}{A}(k_F,\Lambda) = \frac{3\hbar^2}{10m}k_F^2 + \frac{t_0}{4\pi^2}k_F^3 + \frac{T_3}{24\pi^2}k_F^{3+3\alpha} + \frac{\Theta_s}{40\pi^2}k_F^5 + \frac{\Delta E^{(2)}}{A}(k_F,\Lambda).$$

$$\Theta_s = 3t_1 + t_2(5+4x_2)$$

$$\frac{\Delta E_d^{(2)}(k_F,\Lambda)}{A} = -\frac{m}{288\pi^4\hbar^2}\Lambda k_F^3\left[C_0 T_3^2 k_F^{6\alpha} + C_1 T_3 k_F^{2+3\alpha} + C_2 k_F^4\right]$$

$$\frac{\Delta E_d^{(2)}(k_F,\Lambda)}{A} = -\frac{m}{288\pi^4\hbar^2}\Lambda k_F^3 \left[C_0 T_3^2 k_F^{6\alpha} + C_1 T_3 k_F^{2+3\alpha} + C_2 k_F^4\right]$$

- Several possibilities, for example C_1=C_2=0 and α=1/3

- **Second possibility: 6 α = 2+3 α=4 -> α=2/3, with:**

$$C_0^R T_3^{R\,2} + C_1^R T_3^R + C_2^R = 0$$

An EFT way of analyzing the problem…

Redefinition of the parameters

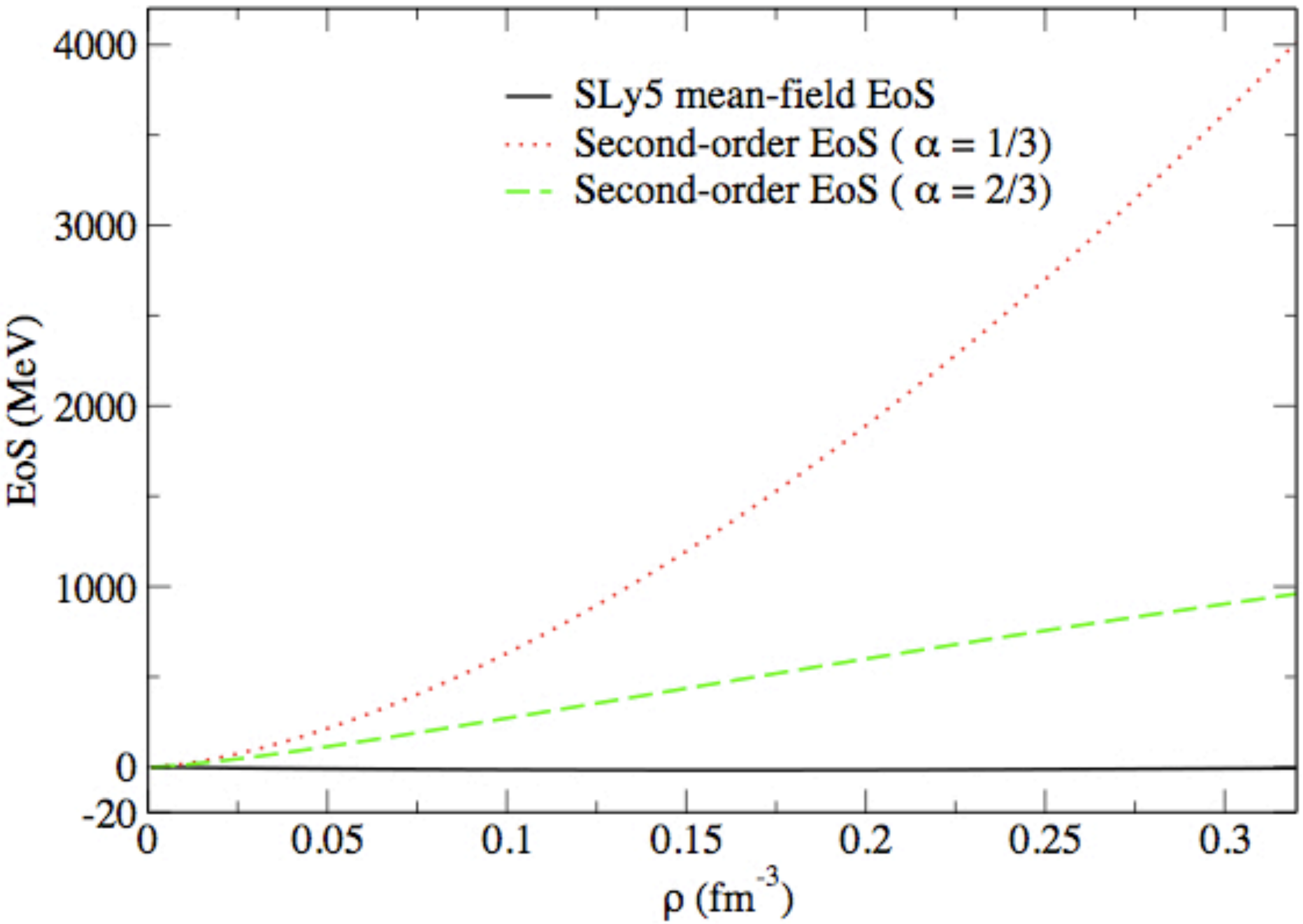
$$t_0^R = t_0(\Lambda) - \frac{m\Lambda}{2\pi^2\hbar^2} B_0(\Lambda),$$

$$T_3^R = T_3(\Lambda) \left[1 - \frac{m\Lambda}{2\pi^2\hbar^2} B_1(\Lambda)\right]$$

$$\theta_s^R = \theta_s(\Lambda) - \frac{m\Lambda}{2\pi^2\hbar^2} B_2(\Lambda),$$

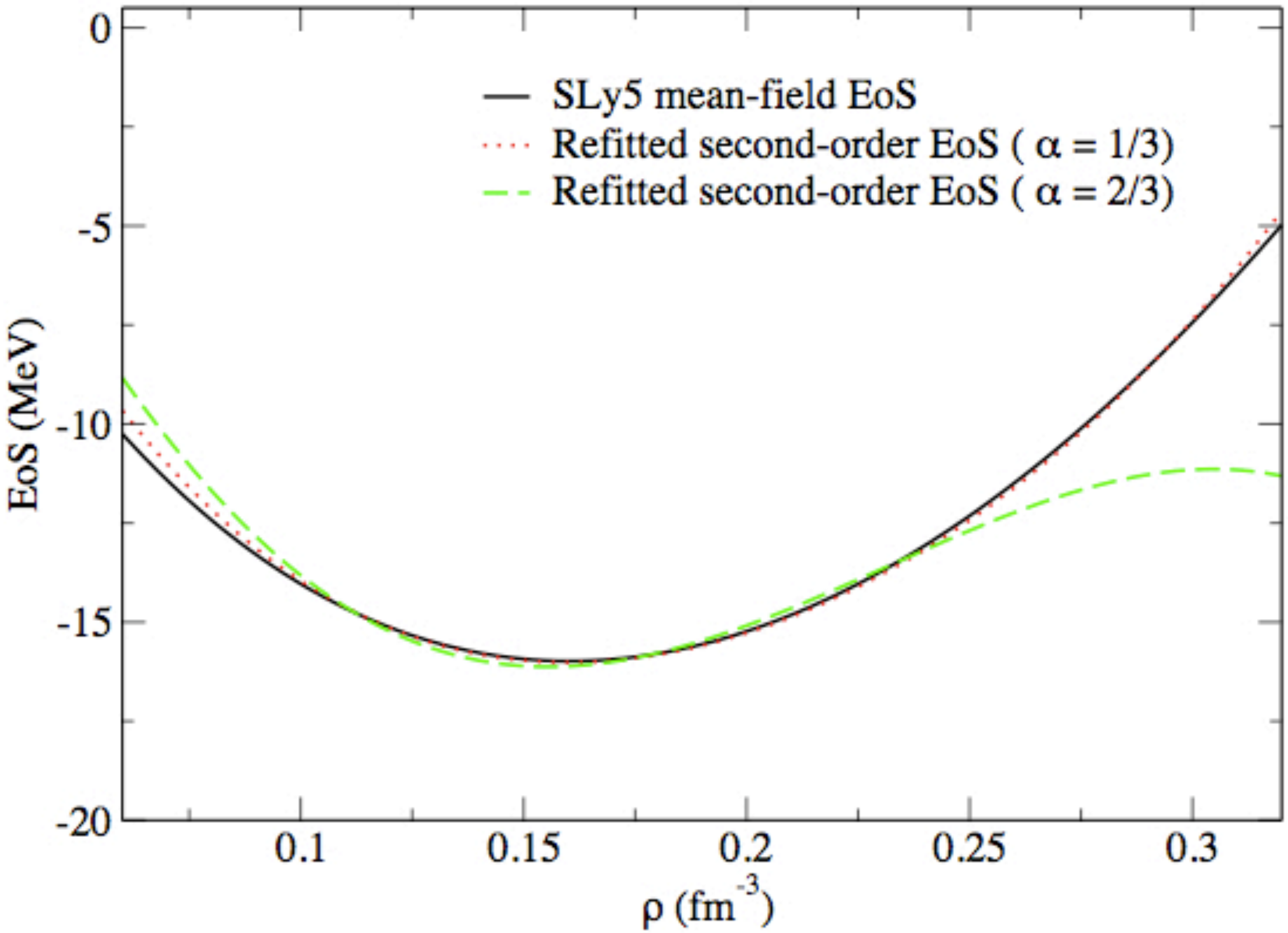
The 'bare' parameters depend on the cutoff so that the renormalized parameters do not depend on the cutoff

How the equation of state looks like:



Unphysical equation of state! … double counting

The parameters are adjusted in the two cases:



To suppress the divergent part:

$$\frac{\Delta E^{(2)}(k_F,\Lambda)}{A} = \frac{\Delta E_f^{(2)}(k_F)}{A} + \frac{\Delta E_a^{(2)}(k_F,\Lambda)}{A} + \frac{\Delta E_d^{(2)}(k_F,\Lambda)}{A},$$

Counter terms have to be added …

Ensuring renormalizabilty is a step towards the more general objective: searching for the correct power counting that indicates the proper hierarchy of allowed interactions

Adding counter terms : analysis of perturbativity (hierarchy)

Beyond mean field in the perturbative many-body problem

2

Zero-range terms in conventional forces -> **ultraviolet divergences** ***when one goes beyond the mean field***

3

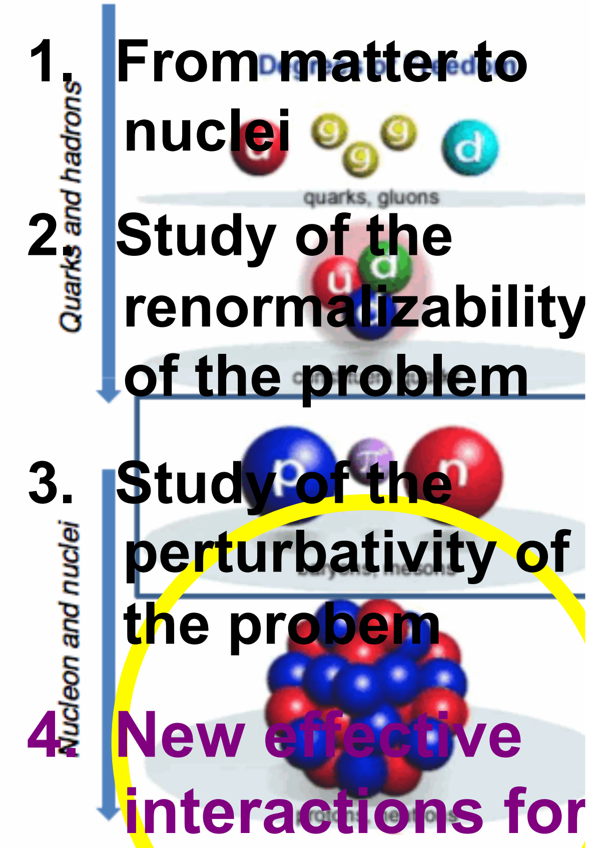
Is our problem really **perturbative** ***when conventional forces are used?***

Regularization techniques

Power counting analysis

Effective field theories (chiral interactions)

Important aspects to be analyzed:



1. From matter to nuclei
2. Study of the renormalizability of the problem
3. Study of the perturbativity of the probem
4. New effective interactions for beyond mean-field theories

APPLICATIONS:

Accurate spectroscopic studies for stable and exotic systems (particle-vibration models)

Accurate analysis of excited states (second RPA)

…